

Game Theory

Thinking Strategically III: Nash Equilibria, Part 2

Contents:
Nash Equilibria
Coordination Games
Sequential Games

Review:
Nash Equilibria

Nash Equilibria

Set of strategies, one for each player, such that no player has an incentive to change their strategy.

Last Time...

Example: Investment Game

Nash Equilibria

Example: Investment Game

Players: you

Strategies: invest \$0 or invest \$10

Payoffs: if you invest \$0, you win/lose nothing
if you invest \$10, win \$11 if >90% invests
win \$0 otherwise.

Nash Equilibria

Example: Investment Game

		2	
		invest	refrain
1	Invest	1, 1	-10, 0
	Refrain	0, -10	0, 0

Nash Equilibria

Example: Investment Game

		2	
		invest	refrain
1	Invest	1, 1	-10, 0
	Refrain	0, -10	0, 0

Nash Equilibria

Example: Bank Run



Nash Equilibria

Example: Investment Game
Communication can help!

		2	
		invest	refrain
1	Invest	1, 1	-10, 0
	Refrain	0, -10	0, 0

Nash Equilibria

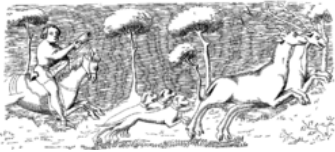
Example: Investment Game
Communication can help!

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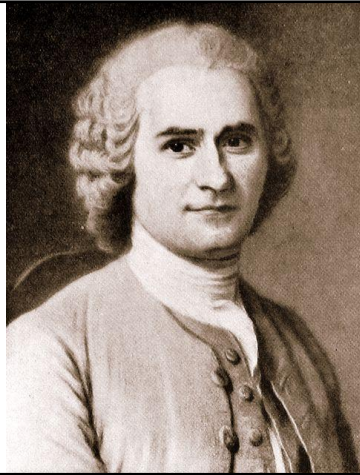
**Example: Stag
Hunt**

Nash Equilibria

Example: Stag Hunt



Jean Jacques Rousseau (1712-1778)
The Discourse on the Origin of Inequality



Nash Equilibria

Example: Stag Hunt

		2	
		stag	hare
1	Stag	3, 3	0, 2
	Hare	2, 0	1, 1

Nash Equilibria

Example: Stag Hunt

		2	
		stag	hare
1	Stag	3, 3	0, 2
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Nash Equilibria

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Nash Equilibria

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Nash Equilibria

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Nash Equilibria

Example: Stag Hunt
Communication can help!

		2	
		stag	hare
1	Stag	3, 3	0, 2
	Hare	2, 0	1, 1

Example: Meeting Game

Nash Equilibria

What about this one?

		2	
		go	stay
1	Go	3, 3	0, 1
	Stay	1, 0	1, 1

Nash Equilibria

		2	
		go	stay
1	Go	3, 3	0, 1
	Stay	1, 0	1, 1

Nash Equilibria

Communication helps!

		2	
		go	stay
1	Go	3,3	0,1
	Stay	1,0	1,1

Example: The Prisoners' Dilemma

Nash Equilibria

Example: The Prisoners' Dilemma
Is there a Nash Equilibrium?

		2	
		c	d
1	C	2,2	0,3
	D	3,0	1,1

Nash Equilibria

Example: The Prisoners' Dilemma
Is there a Nash Equilibrium?

		2	
		c	d
1	C	2,2	0,3
	D	3,0	1,1

Nash Equilibria

Example: The Prisoners' Dilemma

Is there a Nash Equilibrium?

Does communication help?
(e.g., "Let's both choose B!")

Why or why not?

		2	
		c	d
1	C	2, 2	0, 3
	D	3, 0	1, 1

Nash Equilibria

What might help instead?

... What about repeating the game?

		2	
		c	d
1	C	2, 2	0, 3
	D	3, 0	1, 1

Coordination Games

Coordination Game

		2		
		a	b	c
1	A	1, 1	0, 0	0, 0
	B	0, 0	1, 1	0, 0
	C	0, 0	0, 0	1, 1

(This is a *pure coordination game*.)

Coordination Game

		2		
		a	b	c
1	A	1,1	0,0	0,0
	B	0,0	1,1	0,0
	C	0,0	0,0	1,1

Nash Equilibria?

Coordination Game

Communication helps here too.
(But what if you can't?)

		2		
		a	b	c
1	A	1,1	0,0	0,0
	B	0,0	1,1	0,0
	C	0,0	0,0	1,1

Example: Battle of the Wills

Battle of the Wills

Get into pairs.
Decide who is "Player 1" and who is "Player 2".

Battle of the Wills

Get into pairs.

Decide who is "Player 1" and who is "Player 2".

Without talking, write down either "heads" or "tails".

You win if you write down the same thing.

Battle of the Wills

Get into pairs.

Decide who is "Player 1" and who is "Player 2".

Without talking, write down either "heads" or "tails".

You win if you write down the same thing.

Catch: If you both write "heads", Player 1 wins \$2 and Player 2 wins \$1. If you both write "tails", Player 2 wins \$2 and Player 1 wins \$1.

Battle of the Wills

		2	
		h	t
1	H	2, 1	0, 0
	T	0, 0	1, 2

(This is an *impure* coordination game.)

Battle of the Wills

Can communication help here?

How?

		2	
		h	t
1	H	2, 1	0, 0
	T	0, 0	1, 2

(This is an *impure* coordination game.)

Battle of the Wills

According to Jeremy Waldron, games like this are the **key to understanding politics**:

We want to act together in regard to some matter M, but one of us thinks it is important to follow policy X while others think it is important to follow policy Y, and none of us has reason to think any of the others a better judge of the merits of M than himself...

In these circumstances, the following will not be a way of settling on a common policy: each does whatever he thinks is important to do about M. We must find a way of choosing a single policy in which [we] ... can participate despite our disagreements on the merits. [Law and Disagreement, p.107]

		2	
		h	t
1	H	2, 1	0, 0
	T	0, 0	1, 2



Battle of the Wills

What if Player 1 gets to go first?
[First Move Advantage]

		2	
		h	t
1	H	2, 1	0, 0
	T	0, 0	1, 2

(This is an *impure coordination* game.)

Sequential Games

Sequential Games

The players take turns. They can observe each other's moves.

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Example: Here is a game in three rounds. Player A goes first, then Player B, then Player A again.

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Each player can say either "1" or "2" (someone needs to keep track).

Sequential Games

The players take turns. They can observe each other's moves.

Example: Here is a game in three rounds. Player A goes first, then Player B, then Player A again.

Each player can say either "1" or "2" (someone needs to keep track).

If the numbers sum to 5, Player A wins; otherwise, Player B wins.

Sequential Games

The players take turns. They can observe each other's moves.

Who won?

Sequential Games

The players take turns. They can observe each other's moves.

Who won?

Player A? Why?

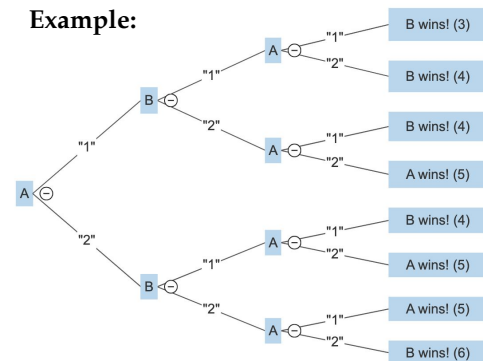
Extensive Form

Extensive Form

Represent games with *trees* rather than *tables*.

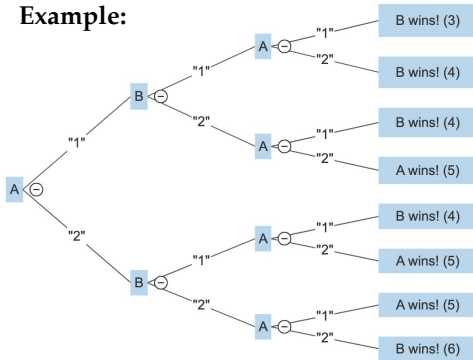
Sequential Games

Example:



Sequential Games

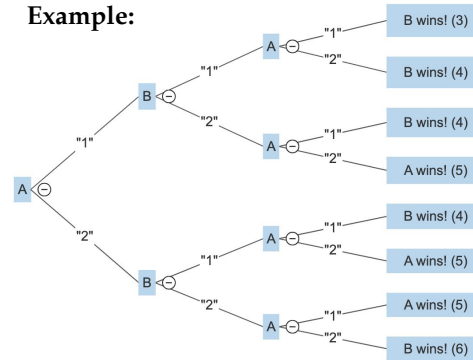
Example:



Choice Nodes

Sequential Games

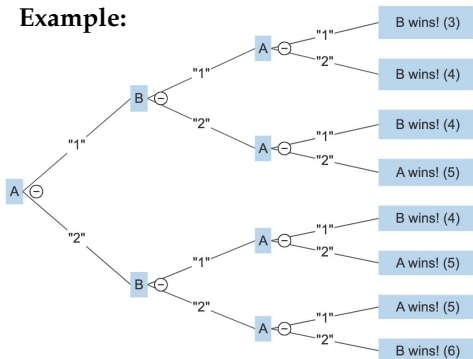
Example:



Choice Nodes
Paths/Strategies

Sequential Games

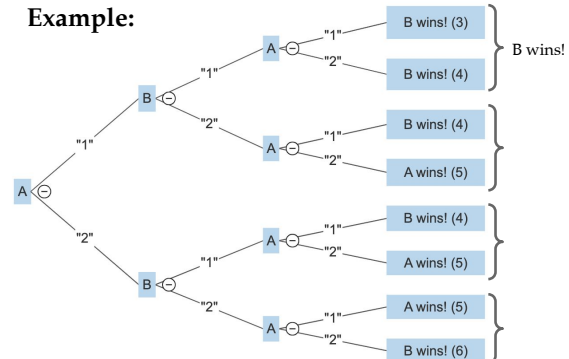
Example:



Choice Nodes
Paths/Strategies
Payoffs

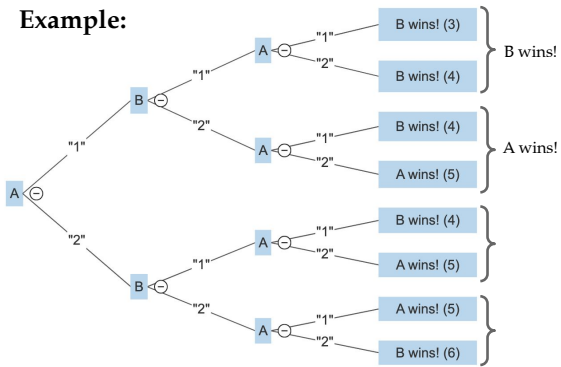
Sequential Games

Example:



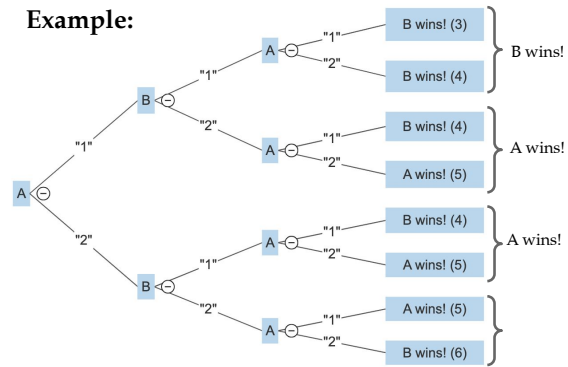
Sequential Games

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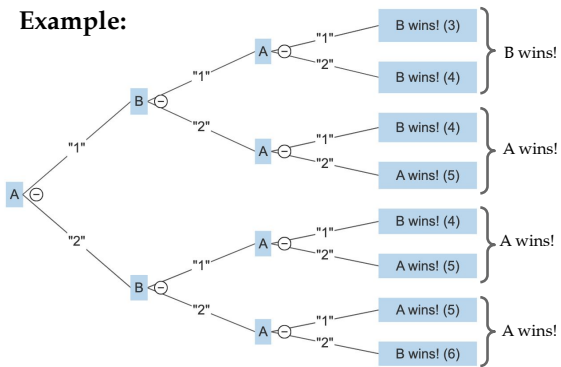
Sequential Games

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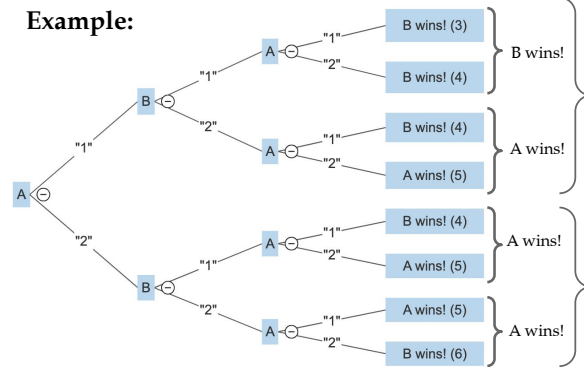
Sequential Games

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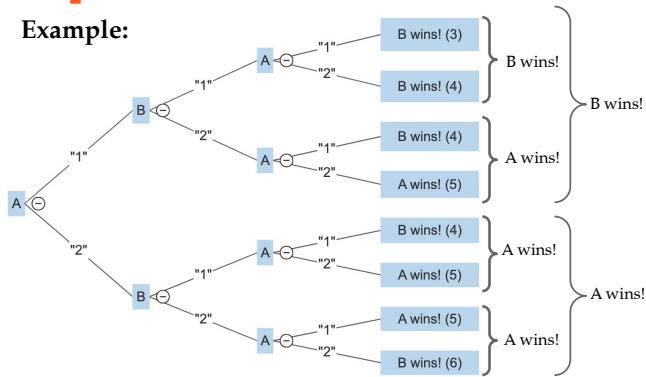
Sequential Games

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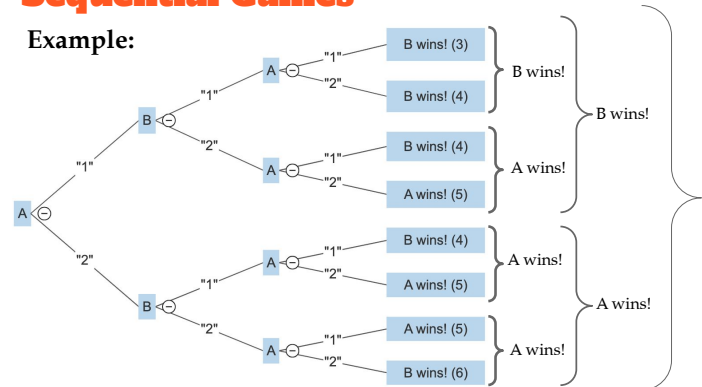
Sequential Games

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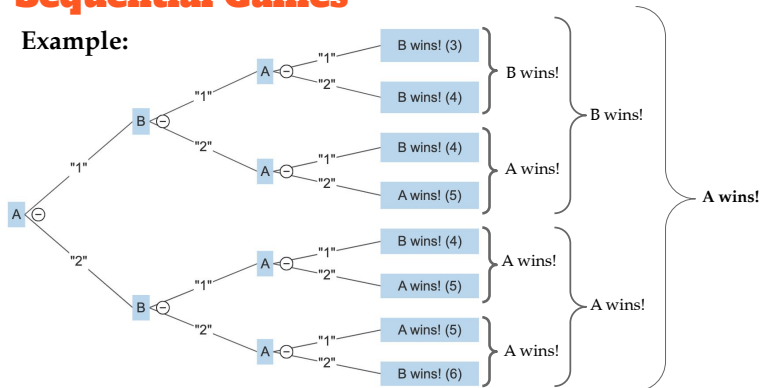
Sequential Games

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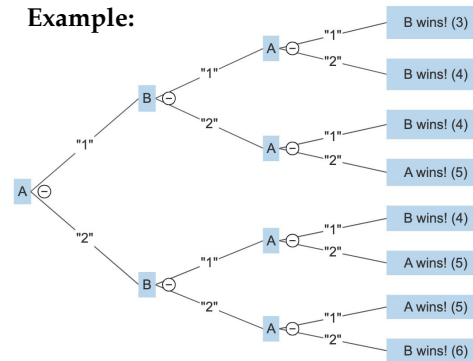
Sequential Games

Example:



Sequential Games

Example:



Player A has a winning strategy:

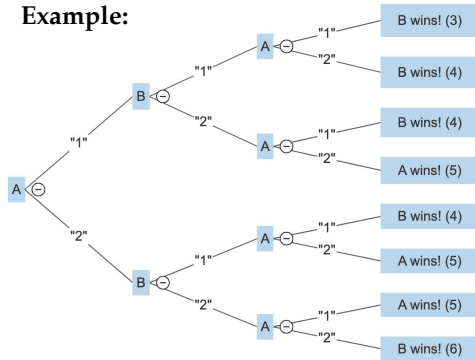
First, say "2".

If Player B says "1", say "2";
If Player B says "2", say "1".

[First Move Advantage]

Sequential Games

Example:



Player A has a winning strategy:

First, say "2".
If Player B says "1", say "2";
If Player B says "2", say "1".

We solved this game using *backwards induction*.

Sequential Games

Example: Dividing the Cake Game.



Sequential Games

Example: Dividing the Cake Game.

I Cut
|
You Choose



Sequential Games

Example: Hungry Lion Game.



Commitment Strategies

Commitment Strategies

Example: Burning Your Ships (and other commitment strategies)



Commitment Strategies

Example: Burning Your Ships (and other commitment strategies)

Hawk/Dove Game

		2	
		h	d
1	H	-1, -1	10, 0
	D	0, 10	5, 5

Commitment Strategies

Example: Burning Your Ships (and other commitment strategies)

Hawk/Dove Game

But Player 2 moves first...

		2	
		h	d
1	H	-1, -1	10, 0
	D	0, 10	5, 5

Commitment Strategies

Example: Burning Your Ships (and other commitment strategies)

Hawk/Dove Game

But Player 2 moves first...

What will happen?

		2	
		h	d
1	H	-1, -1	10, 0
	D	0, 10	5, 5

Commitment Strategies

Example: Burning Your Ships (and other commitment strategies)

Hawk/Dove Game

But Player 2 moves first...

What will happen?

		2	
		h	d
1	H	-1, -1	10, 0
	D	0, 10	5, 5

What could Player 1 do to gain an advantage?

Commitment Strategies

Example: Burning Your Ships (and other commitment strategies)

Player 1 could "burn their ships".

		2	
		h	d
1	H	-1, -1	10, 0
	D	0, 10	5, 5

Commitment Strategies

Example: Burning Your Ships (and other commitment strategies)

Player 1 could "burn their ships":

		2	
		h	d
1	H	-1, -1	10, 0
	D	0, 10	5, 5

Make it so that they cannot play D (even if they wanted to).

Commitment Strategies

Example: The Doomsday Device (from *Dr. Strangelove or: How I Learned to Stop Worrying and Love the Bomb*)



Commitment Strategies

Example: The Doomsday Device (from *Dr. Strangelove or: How I Learned to Stop Worrying and Love the Bomb*)



Questions?